Synthesis and Analysis of an Electric Drive with Sensorless Position Control

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***Abstract*—An approach to sensorless position control of permanent magnet DC motor drives is presented in this paper. The rotor position has been estimated by the respective back EMF voltage, measuring armature current. Using a discrete vector-matrix description of the controlled object, an optimal modal control has been synthesized. Detailed analysis has been carried out by means of mathematical modeling and computer simulation for the transient and steady state regimes. The results obtained show that the applied control method can provide good performance.**

***Index Terms*—DC motor drive, optimal modal control, position control, sensorless control, state observer**

I. INTRODUCTION

Position is one of the main controlled variables in electric drive systems. Movement control of the driven mechanisms is required in many applications, such as: machine tools; lifting machines; woodworking machines; manipulators and robots; antennas; radio telescopes, etc.

Good performance can be provided by a cascade control system, including a non-linear position controller with shifting structure [1]. Such a controller provides for maximum deceleration pace, but approaching the reference position its gain should be limited in accordance with the condition of lack of overshoot. This, on the other hand, leads to some deterioration of the driving system dynamics. A solution to this problem has been suggested in [2], where optimal modal control is applied.

The sensorless control of electric drives reduces hardware costs and improves mechanical reliability. For this reason development of drive systems without sensors for the respective mechanical coordinates is a topical problem of contemporary electric drives theory.

An approach to sensorless position control of DC motor drives is described in this paper. The controlled object consists of a four-quadrant transistor chopper and a permanent magnet DC motor. The rotor position has been estimated indirectly by the respective back EMF voltage, measuring only armature current.

Using a discrete vector-matrix description of the controlled object, a state observer has been synthesized, as well as the respective optimal modal controller applying a complex criterion for optimization [3].

Detailed study has been carried out by means of mathematical modeling and computer simulation for the respective dynamic and static regimes at various loading conditions. The results obtained show that the applied method of control can provide good performance.

II. MODELING OF THE CONTROLLED OBJECT

The vector-matrix model of the DC motor drive under consideration is as follows:

$$\begin{bmatrix} \dfrac{d\theta}{dt} \\ \dfrac{d\omega}{dt} \\ \dfrac{di_a}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \dfrac{K_t}{J} \\ 0 & -\dfrac{K_e}{L_a} & -\dfrac{R_a}{L_a} \end{bmatrix} \begin{bmatrix} \theta \\ \omega \\ i_a \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dfrac{K_c}{L_a} \end{bmatrix} v + \begin{bmatrix} 0 \\ -\dfrac{1}{J} \\ 0 \end{bmatrix} i_l , \quad (1)$$

where: θ is angular position; ω – motor speed; i_a – armature current of the motor; K_e – back EMF voltage coefficient; K_t – torque coefficient; R_a – armature circuit resistance; L_a – armature inductance; K_c – amplifier gain of the chopper; v – input control signal of the chopper; J – total inertia referred to the motor shaft; i_l – armature current which is determined by the respective load torque.

The basic parameters of the controlled object are as follows: $K_e = 0.229$ Vs/rad; $K_t = 0.229$ Nm/A; $R_a = 0.755\,\Omega$; $L_a = 0.003$ H; $K_c = 3.63$; $J = 0.006\,\text{kg.m}^2$.

In order to obtain a suitable simulation model some assumptions have been made, such as:

- the transistor chopper operates at sufficiently high commutation frequency, due to which its delay is neglected e.g. $v_a = K_c v$;
- the load torque is limited, constant and unknown;
- the parameters of the model (1) are constant and known;
- the armature current is measured and the angular position is calculated.

The analogue model of the DC motor drive is realized according to Eq. (1) and it is shown in Fig. 1.

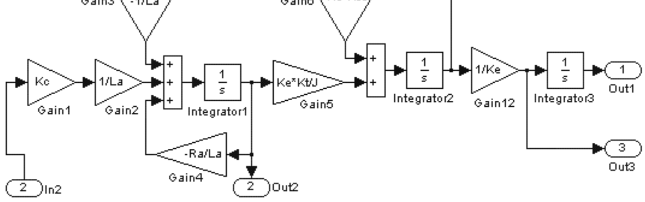

Fig. 1 Model of the controlled DC motor drive

In permanent magnet DC motors the back EMF voltage E is proportional to the motor speed ω :

$$E = K_e \omega = K_e \frac{d\theta}{dt} . \tag{2}$$

As the angular position θ is not measured directly, in this case it can be obtained from the back EMF voltage E. For small quantization periods T_0 Eq. (2) is transformed into the next form:

$$\theta(k) = \theta(k-1) + \frac{T_0}{K_e} E(k) . \tag{3}$$

The back EMF voltage can be calculated on the bases of the armature voltage $v_a = K_c v$ and armature current i_a after its measurement.

The voltage applied on the motor armature is expressed by the equation:

$$v_a = R_a i_a + L_a \frac{di_a}{dt} + E , \tag{4}$$

from where

$$E = v_a - R_a i_a - L_a \frac{di_a}{dt} . \tag{5}$$

For such a small quantization periods T_0 Eq. (5) can be transformed into the next expression:

$$\begin{aligned} E(k) &= v_a(k) - R_a i_a(k) - L_a \frac{i_a(k) - i_a(k-1)}{T_0} = \\ &= v_a(k) - [R_a i_a(k) + \frac{L_a}{T_0} i_a(k) - \frac{L_a}{T_0} i_a(k-1)]. \end{aligned} \tag{6}$$

Taking into consideration Eq. (6), Eq. (3) becomes as follows:

$$\begin{aligned} \theta(k) &= \theta(k-1) + \\ &+ \frac{T_0}{K_e} \{v_a(k) - [R_a i_a(k) + \frac{L_a}{T_0} i_a(k) - \frac{L_a}{T_0} i_a(k-1)]\}. \end{aligned} \tag{7}$$

Based on the Eq. (7) a discrete model of the angular position calculator has been developed and its block diagram is shown in Fig. 2.

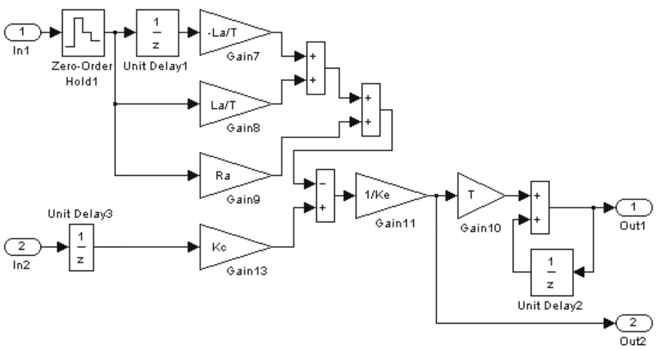

Fig. 2 Model of the angular position calculator

The following notations of state variables have been adopted: $x_1 = \theta$, $x_2 = \omega$, $x_3 = i_a$. The angular position θ can be computed, so in this case it has been assumed that:

$$y(t) = \mathbf{C}\mathbf{x}(t) ,$$

where: $\mathbf{C} = [1 \quad 0 \quad 0]$; $\mathbf{x}^{\mathrm{T}} = [x_1 \quad x_2 \quad x_3]$.

The discrete state-space model of the controlled object can be represented as follows:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} v(k) + \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} i_l . \tag{8}$$

In order to use the quadratic quality criterion in the process of synthesis, the system error of $e(k) = \theta_r(k) - \theta(k)$ should be formulated, where $\theta_r(k)$ is the respective reference input.

It is assumed that both the reference and disturbance inputs are constant, i.e. $\theta_r(k) = \text{const}$ and $i_l = \text{const}$. The following equation concerns the error and state variables, which are not outputs [3]:

$$\begin{bmatrix} x_{1e}(k+1) \\ x_{2e}(k+1) \\ x_{3e}(k+1) \\ x_{4e}(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & a_{11} & -a_{12} & -a_{13} \\ 0 & -a_{21} & a_{22} & a_{23} \\ 0 & -a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_{1e}(k) \\ x_{2e}(k) \\ x_{3e}(k) \\ x_{4e}(k) \end{bmatrix} + \begin{bmatrix} 0 \\ -b_1 \\ b_2 \\ b_3 \end{bmatrix} v_e(k) \tag{9}$$

or

$$\mathbf{x}_e(k+1) = \mathbf{A}_e \mathbf{x}_e(k) + \mathbf{b}_e v_e(k), \ \mathbf{x}_e(0) = \mathbf{x}_{e0},$$

$$k = 0,\ 1,\ 2,\ ...;$$

$$y(k) = \mathbf{C}_e \mathbf{x}_e(k),$$

where:

$$\begin{aligned} x_{1e}(k) &= e(k-1) = \theta_r(k-1) - \theta(k-1); \\ x_{2e}(k) &= e(k) - e(k-1) = -[\theta(k) - \theta(k-1)]; \\ x_{3e}(k) &= \omega(k) - \omega(k-1); \\ x_{4e}(k) &= i_a(k) - i_a(k-1); \\ v_e(k) &= v(k) - v(k-1); \\ \mathbf{C}_e &= [1 \quad 0 \quad 0 \quad 0]. \end{aligned} \tag{10}$$

Eq. (9) has been used for the synthesis of both an optimal modal digital observer and the respective optimal modal controller.

III. SYNTESIS OF THE CONTROL SYSTEM

Synthesis of the digital observer has been realized in accordance with an algorithm presented in [4]. This procedure utilizes the transpositioned additional object [5]:

$$\boldsymbol{\alpha}(k+1) = \mathbf{A}_e^{\mathrm{T}} \boldsymbol{\alpha}(k) + \mathbf{C}_e^{\mathrm{T}} \beta(k) \tag{11}$$

or

$$\begin{bmatrix}\alpha_1(k+1)\\ \alpha_2(k+1)\\ \alpha_3(k+1)\\ \alpha_4(k+1)\end{bmatrix}=\begin{bmatrix}1 & 1 & 0 & 0\\ 0 & a_{11} & -a_{21} & -a_{31}\\ 0 & -a_{12} & a_{22} & a_{32}\\ 0 & -a_{13} & a_{23} & a_{33}\end{bmatrix}\begin{bmatrix}\alpha_1(k)\\ \alpha_2(k)\\ \alpha_3(k)\\ \alpha_4(k)\end{bmatrix}+\begin{bmatrix}1\\0\\0\\0\end{bmatrix}\beta(k). \quad (12)$$

The $\mathbf{A}_e^{\mathrm{T}}$ matrix eigenvalues are determined solving the following equation:

$$\det\left\{\begin{bmatrix}1 & 0 & 0 & 0\\ 1 & a_{11} & -a_{21} & -a_{31}\\ 0 & -a_{12} & a_{22} & a_{32}\\ 0 & -a_{13} & a_{23} & a_{33}\end{bmatrix}-\begin{bmatrix}\chi & 0 & 0 & 0\\ 0 & \chi & 0 & 0\\ 0 & 0 & \chi & 0\\ 0 & 0 & 0 & \chi\end{bmatrix}\right\}=0. \quad (13)$$

For quantization period of $T_0 = 0.001\,\mathrm{s}$ the following eigenvalues are obtained:

$$\chi_1 = 1;\ \chi_2 = 1;\ \chi_3 = 0.9879;\ \chi_4 = 0.7870.$$

In this case there are two undesired roots of the open-loop system ($\chi_1 = 1$ and $\chi_2 = 1$), which must be displaced.

Locations for the closed-loop system roots $\mu_1 = 0.05$ and $\mu_2 = 0.1$ are defined, where χ_1 and χ_2 should be placed. The locations of μ_3 and μ_4 are the same as in the open-loop system, i.e. $\mu_3 = \chi_3$ and $\mu_4 = \chi_4$.

In order to define the observer $\mathbf{H}$ matrix, it is necessary to find the elements of $\mathbf{q}_1$ and $\mathbf{q}_2$ eigenvectors corresponding to χ_1 and χ_2, respectively.

The $\mathbf{q}_1$ eigenvector is obtained solving this system of homogenous algebraic equations:

$$(\mathbf{A}_e - \mathbf{I}\chi_i)\mathbf{q}_i = 0, \text{ for } i = 1. \quad (14)$$

For the elements of both eigenvector $\mathbf{q}_1$ and weight matrix $\mathbf{Q}_1$ the following is obtained:

$$\mathbf{q}_1 = \begin{bmatrix}1\\0\\0\\0\end{bmatrix},$$

$$\mathbf{Q}_1 = \mathbf{q}_1\mathbf{q}_1^{\mathrm{T}} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}.$$

These products are computed:

$$\mathbf{b}_e^{\mathrm{T}}\mathbf{q}_1\mathbf{q}_1^{\mathrm{T}} = [1 \quad 0 \quad 0 \quad 0]$$

and

$$\mathbf{b}_e^{\mathrm{T}}\mathbf{q}_1\mathbf{q}_1^{\mathrm{T}}\mathbf{b}_e = 1.$$

Weight coefficient $r_1 = 0.0554$ and the $\lambda_1 = 1.0526$ coefficient are calculated:

After the first iteration, for the optimal modal feedback gain the following is obtained:

$$\boldsymbol{\gamma}_1 = \begin{bmatrix}-0.95\\0\\0\\0\end{bmatrix}.$$

In order to displace χ_2 to location μ_2, the new system with a state matrix should be optimized:

$$\mathbf{A}_e^c = \mathbf{A}_e + \mathbf{b}_e\boldsymbol{\gamma}_1.$$

The $\mathbf{q}_2$ eigenvector is derived after solving the system of homogeneous algebraic equations:

$$(\mathbf{A}_e^c - \mathbf{I}\chi_i)\mathbf{q}_i = 0 \quad за \quad i = 2. \quad (15)$$

For the elements of both the eigenvector $\mathbf{q}_2$ and weight matrix $\mathbf{Q}_2$ respectively, the following is obtained:

$$\mathbf{q}_2 = \begin{bmatrix}0.7250\\0.6887\\0\\0\end{bmatrix};$$

$$\mathbf{Q}_2 = \mathbf{q}_1\mathbf{q}_1^{\mathrm{T}} = \begin{bmatrix}0.5256 & 0.4993 & 0 & 0\\ 0.4993 & 0.4744 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}.$$

The next products are computed:

$$\mathbf{b}_e^{\mathrm{T}}\mathbf{q}_1\mathbf{q}_1^{\mathrm{T}} = [0.5256 \quad 0.4993 \quad 0 \quad 0]$$

and

$$\mathbf{b}_e^{\mathrm{T}}\mathbf{q}_1\mathbf{q}_1^{\mathrm{T}}\mathbf{b}_e = 0.5256.$$

The respective weight coefficient $r_2 = 0.0649$ and the $\lambda_2 = 1.1111$ coefficient are computed.

After the second iteration, the optimal modal feedback gain is obtained:

$$\boldsymbol{\gamma}_2 = \begin{bmatrix}-0.900\\-0.855\\0\\0\end{bmatrix}.$$

Since in this case there are two undesired values ($\chi_1 = 1$ и $\chi_2 = 1$), the optimal modal feedback gain becomes:

$$\boldsymbol{\gamma}^{*\mathrm{T}} = \boldsymbol{\gamma}_1^{\mathrm{T}} + \boldsymbol{\gamma}_2^{\mathrm{T}} = [-1.8500 \quad -0.8550 \quad 0 \quad 0].$$

The observer feedback vector is formulated:

$$\mathbf{H} = \begin{bmatrix}h_1\\h_2\\h_3\\h_4\end{bmatrix} = \gamma^* = \begin{bmatrix}-1.850\\-0.855\\0\\0\end{bmatrix}.$$

The observer equation is as follows [4]:

$$\hat{\mathbf{x}}_e(k+1) = \mathbf{A}_e\hat{\mathbf{x}}_e(k) + \mathbf{b}_e\mathbf{u}_e(k) + \mathbf{H}\Delta\mathbf{e}(k) =$$
$$= \mathbf{A}_e\hat{\mathbf{x}}(\mathbf{k}) + \mathbf{b}_e\mathbf{u}_e(k) + \mathbf{H}[y(k) - \mathbf{C}\hat{\mathbf{x}}(\mathbf{k})]$$

or

$$\begin{bmatrix}\hat{x}_{1e}(k+1)\\ \hat{x}_{2e}(k+1)\\ \hat{x}_{3e}(k+1)\\ \hat{x}_{4e}(k+1)\end{bmatrix} = \begin{bmatrix}1 & 1 & 0 & 0\\ 0 & a_{11} & -a_{12} & -a_{13}\\ 0 & -a_{21} & a_{22} & a_{23}\\ 0 & -a_{31} & a_{32} & a_{33}\end{bmatrix} \mathrm{x}$$
$$\mathrm{x} \begin{bmatrix}\hat{x}_{1e}(k)\\ \hat{x}_{2e}(k)\\ \hat{x}_{3e}(k)\\ \hat{x}_{4e}(k)\end{bmatrix} \begin{bmatrix}0\\ -b_1\\ b_2\\ b_3\end{bmatrix} u_e(k) + \begin{bmatrix}h_1\\ h_2\\ h_3\\ h_4\end{bmatrix}\Delta e(k), \tag{16}$$

where $\Delta e(k) = \mathbf{y}(k) - \mathbf{C}\hat{\mathbf{x}}(k)$.

These equations produce the state variables valuation. Based on them a model of the optimal modal observer has been developed and its block diagram is shown in Fig.3.

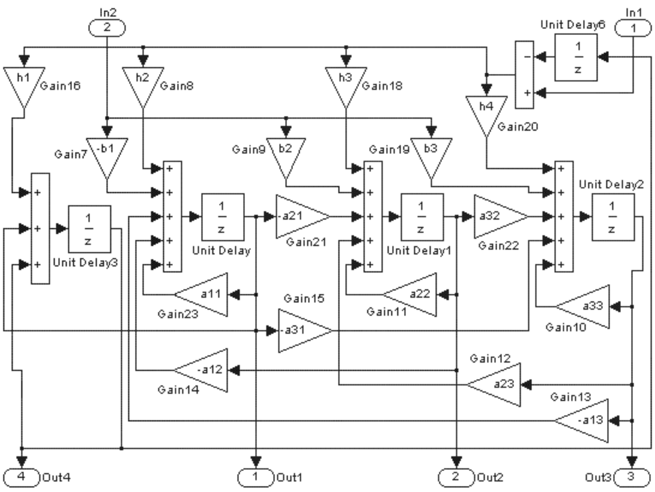

Fig. 3 Model of the optimal modal observer

Synthesis of the optimal modal controller has been realized by an algorithm described in [3]. In this case synthesis is carried out based on Eq. (9).

At quantization period of $T = 0.001\,\mathrm{s}$ for the matrix $\mathbf{A}_e$ eigenvalues, the following is obtained:

$$\chi_1 = 0.7870;\ \chi_2 = 0.9879;\ \chi_3 = 1;\ \chi_4 = 1.$$

Among these values two undesired roots exist ($\chi_3 = 1$ and $\chi_4 = 1$), which should be displaced.

Locations for the closed-loop system roots $\mu_3 = 0.9$ and $\mu_4 = 0.8$ are defined, where χ_3 and χ_4 should be placed. The locations of μ_1 and μ_2 are the same as in the open-loop system, i.e. $\mu_1 = \chi_1$ and $\mu_2 = \chi_2$.

In order to determine the optimal modal controller matrix $\mathbf{K}$, it is necessary to find the elements of the eigenvector $\mathbf{q}_3$, corresponding to χ_3, as well as the eigenvector $\mathbf{q}_4$, corresponding to χ_4.

The $\mathbf{q}_4$ eigenvector is obtained after solving the following system of homogeneous algebraic equations:

$$(\mathbf{A}_e^{\mathrm{T}} - \mathbf{I}\chi_{\mathrm{i}})\mathbf{q}_{\mathrm{i}} = 0,\ \text{for } i = 4. \tag{17}$$

The elements of eigenvector $\mathbf{q}_4$ and weight matrix $\mathbf{Q}_4$ are obtained as follows:

$$\mathbf{q}_4 = \begin{bmatrix}0.0000\\ -0.9356\\ 0.3529\\ 0.0123\end{bmatrix},$$

$$\mathbf{Q}_4 = \begin{bmatrix}0.0000 & 0.0000 & 0.0000 & 0.0000\\ 0.0000 & 0.8753 & -0.3302 & -0.0115\\ 0.0000 & -0.3302 & 0.1245 & 0.0043\\ 0.0000 & -0.0115 & 0.0043 & 0.0002\end{bmatrix}.$$

Next products are calculated:

$$\mathbf{b}_e^{\mathrm{T}}\mathbf{Q}_4 = [0.0000\ \ -0.0139\ \ 0.0052\ \ 0.0002]$$

and

$$\mathbf{b}_e^{\mathrm{T}}\mathbf{Q}_4\mathbf{b}_e = 2.1994 \times 10^{-4}.$$

For these coefficients the following values are obtained:

$$r_4 = 0.0044;\ \lambda_4 = 5.$$

The optimal modal feedback gain is determined:

$$\boldsymbol{\gamma}_1 = \begin{bmatrix}0.0000\\ 12.6171\\ -4.7594\\ -0.1653\end{bmatrix}.$$

In order to displace χ_3 to location μ_3, the new system with a state matrix should be optimized:

$$\mathbf{A}_e^{\mathrm{c}} = \mathbf{A}_e + \mathbf{b}_e\boldsymbol{\gamma}_1.$$

The $\mathbf{q}_3$ eigenvector is obtained after solving the system of homogeneous algebraic equations:

$$(\mathbf{A}_e^{c\,\mathrm{T}} - \mathbf{I}\chi_{\mathrm{i}})\mathbf{q}_{\mathrm{i}} = 0,\ \text{for } i = 3. \tag{18}$$

For the elements of eigenvector $\mathbf{q}_3$ and weight matrix $\mathbf{Q}_3$ respectively, the following is obtained:

$$\mathbf{q}_3 = \begin{bmatrix}-0.0110\\ -0.9994\\ 0.0331\\ 0.0006\end{bmatrix},$$

$$\mathbf{Q}_3 = \mathbf{q}_3\mathbf{q}_3^{\mathrm{T}} = \begin{bmatrix}0.0001 & 0.0110 & -0.0004 & 0.0000\\ 0.0110 & 0.9988 & -0.0331 & -0.0006\\ -0.0004 & -0.0331 & 0.0011 & 0.0000\\ 0.0000 & -0.0006 & 0.0000 & 0.0000\end{bmatrix}.$$

The products are defined:

$$\mathbf{b}_e^{\mathrm{T}}\mathbf{q}_1\mathbf{q}_2^{\mathrm{T}} = [-0.0096 \quad -0.871 \quad 0.0288 \quad 0.0005] \times 10^{-3}$$

and

$$\mathbf{b}_e^{\mathrm{T}}\mathbf{q}_1\mathbf{q}_2^{\mathrm{T}}\mathbf{b}_e = 7.5961 \times 10^{-7}.$$

For these coefficients the following values are obtained: $r_3 = 6.8365 \times 10^{-5}$ and $\lambda_3 = 10$.

At the second iteration, the optimal modal feedback gain becomes:

$$\boldsymbol{\gamma}_2 = \begin{bmatrix} 1.2617 \\ 114.6677 \\ -3.7945 \\ -0.0661 \end{bmatrix}.$$

Since there are two undesired values ($\chi_3 = 1$ and $\chi_4 = 1$), the optimal modal feedback gain is:

$$\boldsymbol{\gamma}^{*\mathrm{T}} = \boldsymbol{\gamma}_1^{\mathrm{T}} + \boldsymbol{\gamma}_2^{\mathrm{T}} = [1.2617 \quad 127.2848 \quad -8.5539 \quad -0.2314].$$

The feedback vector obtains this form:

$$\mathbf{K} = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} = \boldsymbol{\gamma}^* = \begin{bmatrix} 1.2617 \\ 127.2848 \\ -8.5539 \\ -0.2314 \end{bmatrix}$$

and control of the following type is formulated:

$$v_e(k) = \mathbf{K}^{\mathrm{T}}\mathbf{x}_e(k) = k_1x_{1e} + k_2x_{2e} + k_3x_{3e} + k_4x_{4e}. \quad (19)$$

After substitution of $v_e(k)$ in Eq. (10), for the optimal modal controller this expression is obtained:

$$v(k) = v(k-1) + k_1x_{1e} + k_2x_{2e} + k_3x_{3e} + k_4x_{4e}. \quad (20)$$

Analyzing Eq. (20) it can be seen, that the optimal modal controller includes an integral component in its structure. This means that when the driven mechanism is far from the reference position, the integral component would increase at each controlling cycle. It will quickly bring to saturation of the control loop and as a result, the motor will be supplied with maximum voltage. When the mechanism approaches the reference position, the integral component will continue to increase and will become the dominant part of the control signal, forcing the electric drive to exceed the set position.

To solve this problem it is necessary to provide the following condition: when the driven mechanism enters some preliminary set range $(\Delta\theta_s = \theta_r - \theta)$, tcontrol signal is established to the maximum admissible value of $v_{\max}$, after which this error of $\Delta\theta_s$ is processed.

Based on these considerations, as well as on Eq. (20), the model of an optimal modal controller has been constructed. It is represented in Fig. 4.

In the developed system an overtaking current limitation has been applied. The respective function is as follows:

$$v_{cl}(k) = v_i + K_s\omega(k), \quad (21)$$

where: v_i is the current limitation initial code; K_s – scale coefficient.

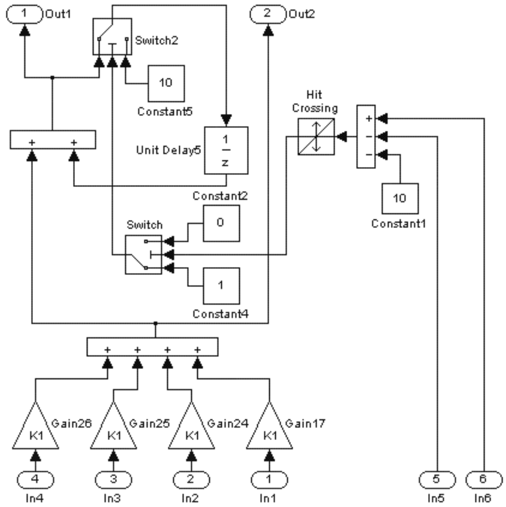

Fig. 4 Model of the optimal modal controller

Hence, the control condition in the presence of current limitation will be:

$$v_c(k) = \begin{cases} v(k) & \text{for} \quad v(k) \le v_{cl}(k); \\ v_{cl}(k) & \text{for} \quad v(k) > v_{cl}(k). \end{cases} \quad (22)$$

In real systems the limitation applied to the control signal should also be taken into account:

$$v_{cr}(k) = \begin{cases} v_c(k) & \text{for} \quad v_c(k) \le v_{\max} \\ v_{\max} & \text{for} \quad v_c(k) > v_{\max}, \end{cases} \quad (23)$$

where $v_{\max}$ is maximum value of the control signal.

The controlling code, which should be used to the chopper control scheme, is determined by conditions (21), (22) and (23). In accordance with these equations an armature current limitation model is composed, shown in Fig. 5.

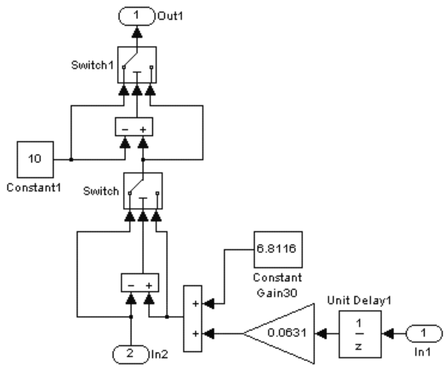

Fig. 5 Model of the current limitation

Practically, the optimal modal control in this case is achieved through consequent realization of Eqs. (19), (20), (21), (22) and (23).

IV. SIMULATION AND PERFORMANCE ANALYSIS

To prove the offered control algorithm functionality some computer simulation models have been developed, using the MATLAB/SIMULINK software package.

The block diagram of the complete drive system with sensorless position control is presented in Fig. 6, where the following notations are used: Subsystem 1 is the controlled object; Subsystem 2 – the angular position calculator; Subsystem 3 – the optimal modal observer; Subsystem 4 – the optimal modal controller; Subsystem 5 – the current limitation.

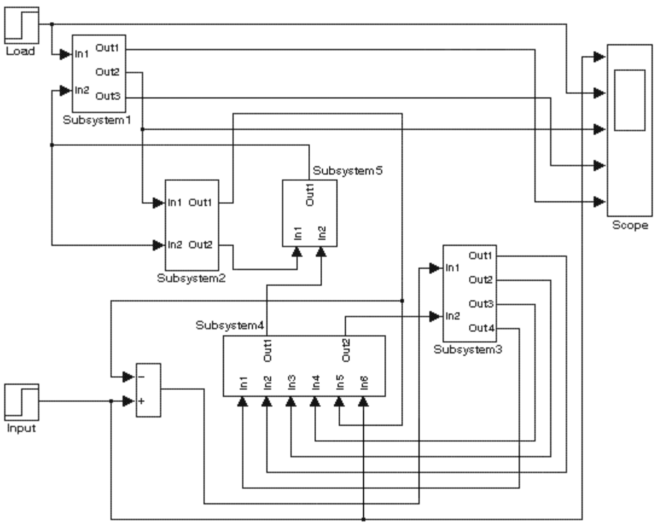

Fig. 6 Model of the complete drive system

Fig. 7 shows some simulation results illustrating the drive system performance for a positioning cycle.

In this case the reference angular position is $\theta_r = 200$ rad and the reference static current is equal to the rated value of $I_{lr} = I_{\text{rat}}$. The motor speed is limited to the rated value of $\omega = \omega_{\text{rat}}$.

During the starting regime the armature current is limited to the maximum admissible value of $I_{\max} = 2.5 I_{\text{rat}}$, which provides good dynamics of the driving system.

The applied quantization period is $T_0 = 0.001\,\text{s}$. The rated data of the used permanent magnet DC motor are as follows: $V_{\text{rat}} = 30\,\text{V}$, $I_{\text{rat}} = 13.1\,\text{A}$, $\omega_{\text{rat}} = 115.19\,\text{rad/s}$.

V. CONCLUSION

An approach to position control of permanent magnet DC motor drives is described in this paper.

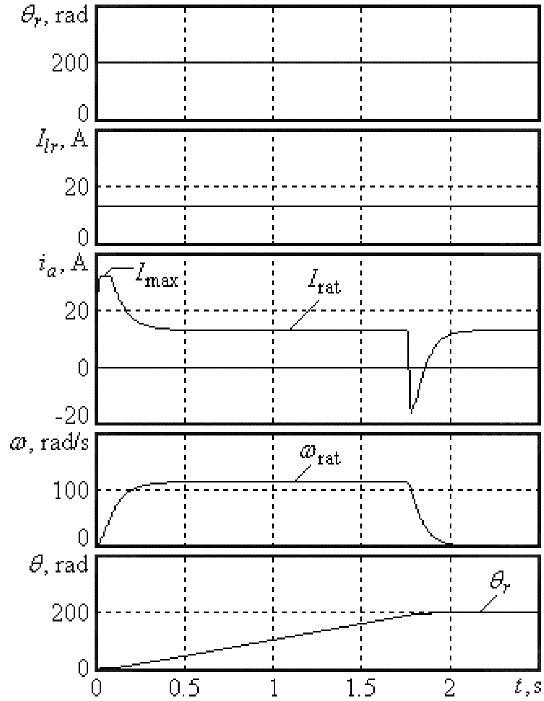

Fig. 7 Time-diagrams for a positioning cycle

Using a discrete vector-matrix description of the controlled object, an optimal modal state observer has been synthesized, as well as an optimal modal controller.

Detailed study has been carried out by means of mathematical modeling and computer simulation for the respective transient and steady state regimes at various loading conditions.

The analysis shows that the represented control method provides good performance, which makes it suitable for a variety of applications.

The simulation models developed as well as the results obtained could be used in the design of such types of positioning systems.

REFERENCES

[1] M. R. Mikhov, "Performance analysis of a positioning electric drive system", Proceedings of the ICEST, pp. 316-319, Sofia, July, 2006.

[2] Ts. T. Georgiev, M. R. Mikhov, "Performance Improvement of a Class of Position Drives", Proceedings of the ICMaS, Vol. 3, pp. 189-194, Bucharest, November, 2008.

[3] Ts. T. Georgiev, "Synthesis of optimal modal discrete controllers", Proceedings of the ICAI, Vol. 2, pp. 119-122, Sofia, October. 2003.

[4] Ts. T. Georgiev, D. G. Genov, "Synthesis of optimal modal observers, Computer Science and Technologies", Vol. 4, No. 1/2, pp. 51-57, Technical University of Varna, Varna, 2006.

[5] R. Iserman. Digital Control Systems, Springer-Verlag, New York, 1981.